A Dirichlet Generalized Ordered Logit Analysis of Women's Labor Supply after Childbirth

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Abstract

This paper investigates the effect of job continuity on the supply of labor of new mothers for the five-year period after their first childbirth. Using the National Longitudinal Survey of Youth 1997 (NLSY97), we extend the beta-logistic model of Heckman and Willis (1977) to a higher-dimensional Dirichlet generalized ordered logit (DGOL) model. This approach considers individual-specific state dependence and heterogeneity in employment choices. The DGOL model nearly doubles the predictive accuracy of the standard generalized ordered logit model. Our study finds that working full-time in any given period increases the likelihood of continuing full-time work in the next year by 65 percent. The findings also suggest the importance of job continuity: a higher job continuity probability increases full-time employment rates by over 10 percent, offsetting potential drawbacks of lower education and older age.

Keywords: labor supply, job continuity, Dirichlet distribution, ordered logit, state dependence, unobserved heterogeneity, structural model
JEL Classification:

1 Introduction

Labor force participation rates of women decline after the birth of their first child (Bianchi 2000; Gornick and Meyers 2003). New mothers may need to balance career with increased domestic responsibility. Maternity leave legislation can help with this task by replacing lost wages and salary and by providing job continuity, which maintains firm-specific human capital. This study investigates how job continuity affects future labor market outcomes for new mothers. We take into account that labor supply displays significant persistence at the individual level (Heckman and Willis 1977; Heckman 1981; Booth, Jenkins, and Serrano 1999; Hyslop 1999; Francesconi 2002; Seetharaman 2004; Haan 2010; Jia and Vattø 2021).

Persistence in labor market outcomes may arise because of true state dependence, where an individual's past employment outcomes determine current labor supply. Observed persistence may also arise because of unobserved heterogeneity, also called spurious state dependence, where the presence of factors not observed in the data determine labor force participation through time. Ignoring the presence of persistence in labor market outcomes may lead to erroneous inferences about the population and faulty design of policy[1] (Prowse 2012).

Observed outcomes (Figure 1) in the National Longitudinal Survey of Youth 1997 (NLSY97) show that, in the year after the birth of the first child, 40 percent of women work full-time, 40 percent work part time, and 20 percent do not work. The data show that 80 percent of new mothers who worked full-time in the year after the birth of their first child continued to work full-time in the second year after the birth, 56 percent who worked part-time in the first year also worked part-time in the second year, and 68 percent who did not work in the first year also did not work in the second year.

[1]The United States is the only member of the Organisation for Economic Co-operation and Development (OECD) that does not have a national paid family leave program. As of now, nine states and the District of Columbia have established or are on the path to implement paid family and medical leave programs. Nonetheless, barriers to access these benefits remain. A few states have strict eligibility criteria based on work tenure, hours worked, or the size of the employer. Some states offer limited benefits that don't sufficiently address the needs of low-wage workers. Several state programs do not provide robust job protection for paid family leave. Consequently, job protection after childbirth may assist career-oriented women in returning to work and accumulating more human capital.

To capture the impact of unobserved heterogeneity, our empirical analysis builds on the beta-logistic framework developed by Heckman and Willis (1977), who study the two choices of participating and not participating. Since the balancing of career and care of the child may involve an intermediate choice, we construct a Dirichlet generalized ordered logit (DGOL) model to be able to include three ordered choices: full-time work, part-time work, and no work. We hypothesize that job continuity for working new mothers is an important determinant of these labor market outcomes. Bailey, Byker, Patel, and Ramnath (2019) analyze the 2004 California Paid Family Leave Act (PFLA) and find little evidence that PFLA increased women's employment, wage earnings, or attachment to employers. For new mothers, taking up PFLA reduced employment by 7 percent and lowered annual wages by 8 percent six to ten years after giving birth. The absence of job protection within the legislation (Isaacs, Healy, and Peters 2017) is a possible explanation for this. Women who give up their jobs after childbirth must take the time to find satisfactory re-employment before they can continue with their careers. Failure to do so may result in these new mothers dropping out of the labor force entirely.

In a pre-sample analysis we estimate the probabilities of returning to the same employer and returning to a different employer within the first year after childbirth. We use these estimates in a DGOL model of choosing to work full-time, part-time, or not at all for the five subsequent years. To preview the results, our findings indicate that engaging in full-time work in any period increases the likelihood of maintaining full-time employment in the following year by 65 percent. Our study also reveals that maintaining job continuity within the first year after childbirth significantly increases full-time employment rates by over 10 percent. Additionally, full-time employment rates for low-educated mothers and older mothers are lower than the average population. Nevertheless, a higher probability of returning to previous employer mitigates these adverse effects on their full-time employment.

Compared to the existing literature, our study makes four primary contributions. First, we extend the consideration of state dependence and unobserved heterogeneity to a high-dimensional framework for ordered choices. Second, we apply this newly-developed methodological approach to evaluate women's labor force participation after childbirth and the connection between job continuity and employment decisions, addressing issues of reverse causality and correlated unobserved heterogeneity. Our DGOL model reveals significant unobserved heterogeneity among women following childbirth resulting in substantial state dependence in employment statuses. Additionally, the DGOL significantly outperforms traditional models like generalized ordered logit and ordered logit in predictive accuracy, with an accuracy rate approximately twice as high as those of the latter two models. Third, given the impracticality of conducting random experiments, our structural model allows us to separately identify the causal impact of previous employment on current employment decisions, thus complementing the reduced-form literature. We find that engaging in full-time work in any period enhances the probability of continuing full-time employment in next year by 65 percent. Finally, we use the estimated parameters to simulate a variety of policy counterfactuals in different scenarios and for various groups of mothers. These policy counterfactuals shed light on the importance of existing maternity leave, paid family leave policies, and corresponding job protection policies.

The remainder of this figure is organized as follows. Section 2 presents the econometric theory. Section 3 presents the data. The empirical results are given in section 4 which also includes counterfactual experiments on the effect of job continuity on women's subsequent employment. Conclusions are given in section 5.

2 Methodology

2.1 The Ordered Logit and Generalized Ordered Logit Models

Techniques such as ordinary least squares regression require that outcome variables have interval or ratio level measurement. The most exact measurement of labor force participation is hours of work, which is a ratio level variable. Because ratio of values are meanful - working 1000 hours per year means working twice as much as 500 hours per year. For our study, we would prefer to use desired hours of work rather than actual hours of work. For example, shorter period of non-employment or unemployment should not distinguish the level of labor force attachment. When the outcome variable is ordinal, the most popular method is the ordered logit model, which is also known as the proportional odds model (Williams 2016).

In mathematical terms, let Y represent an ordinal dependent variable with J levels of labor force participation. The Ordered logit model assumes that Y changes when an unobserved continuous latent variable, Y^* crosses certain thresholds. In our study, we categorize women's employment status as full-time ($Y_i = 3$), part-time ($Y_i = 2$), or not working ($Y_i = 1$), underpinned by the latent variable of their utility, which is determined by their anticipated working hours and characteristics. So, we have three levels ($J = 3$) and two thresholds:

$$\mathrm{P}\left(Y_i > j\right) = \frac{\exp\left(\alpha_j + z_i'\beta\right)}{\left(1 + \exp\left(\alpha_j + z_i'\beta\right)\right)},\ j = 1, 2 \tag{1}$$

where z_i is a vector of covariates. Using equation (1), we can write three probabilities as:

$$\begin{aligned} P(Y = 1) &= \frac{1}{\left(1 + \exp\left(\alpha_1 + z_i'\beta\right)\right)} \\ P(Y = 2) &= \frac{\exp\left(\alpha_1 + z_i'\beta\right)}{\left(1 + \exp\left(\alpha_1 + z_i'\beta\right)\right)} - \frac{\exp\left(\alpha_2 + z_i'\beta\right)}{\left(1 + \exp\left(\alpha_2 + z_i'\beta\right)\right)} \\ P(Y = 3) &= \frac{\exp\left(\alpha_2 + z_i'\beta\right)}{\left(1 + \exp\left(\alpha_2 + z_i'\beta\right)\right)} \end{aligned} \tag{2}$$

The ordered logit model relies on a strong assumption known as the parallel lines or proportional odds assumption. This assumption dictates that the relationship between the predictor variables and the ordinal response variable is consistent across all levels of the response variable. In other words, the coefficient vector is assumed to be the constant across all thresholds.

Violating this assumption could lead to inconsistnet estimates. To illustrate this with our labor-force participation example, one could conceptualize two binary logit submodels: 1) 'no work' versus 'some work', and 2) 'less than full-time' versus 'full-time'. The ordered logit model assumes that the odds ratios are equal across these submodels. But in reality, the odds ratios could differ between these scenarios. For example, the level of education might have a different influence on transition from 'no work' to 'some work' and moving from 'than when full-time' to 'full-time' employment.

The generalized ordered logit model relaxes the parallel lines assumption. The proportional odds are allowed to vary across submodels. The formulation of this model is given by:

$$\mathrm{P}\left(Y_i > j\right) = \frac{\exp\left(\alpha_j + z_i'\beta_j\right)}{\left(1 + \exp\left(\alpha_j + z_i'\beta_j\right)\right)}, \; j = 1, 2. \tag{3}$$

A test devised by Brant (1990) can be used to determine whether the observed deviations from what the proportional odds model predicts are larger than what can be attributed to chance alone.

2.2 The Dirichlet generalized ordered logit Model

Although the generalized ordered logit model relaxes the parallel lines assumption, it still has some limitations and drawbacks. Both the ordered logit and generalized ordered logit models assume that the odds are homogeneous within the population for a given set of covariate values. If the homogeneity of odds assumption is violated, it can lead to inconsistent estimates.

One remedy is to consider higher order moments. Let the probabilities of women's labor force participation of no work, part-time job and full-time job be π_1, π_2, π_3 with $\pi_1+\pi_2+\pi_3 = 1$. Then the probabilities that a woman does not work for x_1 years, works part time for x_2 years. and works full time for x_3 years out of total T years are:

$$P\left(x_1, x_2, x_3, T\right) = \frac{T!}{x_1!x_2!x_3!}\pi_1^{x_1}\pi_2^{x_2}\pi_3^{x_3} \quad . \tag{4}$$

The expected probability of women's labor supply, taking into account their prior labor force participation is:

$$E\left[P\left(x_1, x_2, x_3, T\right)\right] = \int \frac{T!}{x_1!x_2!x_3!}\pi_1^{x_1}\pi_2^{x_2}\pi_3^{x_3} f(\boldsymbol{\pi})d\boldsymbol{\pi} \quad , \tag{5}$$

where $f(\boldsymbol{\pi})$ represents the density function of $(\pi_1,\ \pi_2,\ \pi_3)$, In this work, we assume $f(\boldsymbol{\pi})$ is theDirichlet distribution density:

$$f(\boldsymbol{\pi}) = \frac{1}{B(\alpha_1, \alpha_2, \alpha_3)}\pi_1^{\alpha_1-1}\pi_2^{\alpha_2-1}\pi_3^{\alpha_3-1} \quad , \tag{6}$$

where the parameter vector $(\alpha_1, \alpha_2, \alpha_3)$ has positive components. $B(\alpha_1, \alpha_2, \alpha_3) = \frac{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)}{\Gamma(\alpha_1+\alpha_2+\alpha_3)}$ is the beta function, and $\Gamma(\cdot)$ is the gamma function with $\Gamma(\alpha) = \int_0^1 t^{\alpha-1}e^{-t}dt$.

The Dirichlet distribution is an attractive choice of functional form for the responding probability of $(\pi_1,\ \pi_2,\ \pi_3)$ for several reasons. First, the range of the Dirichlet distribution is from 0 to 1. Second, it has relatively few parameters, one for each of the random components. The parameters $\alpha_1,\ \alpha_1,\ \alpha_3$ govern the shape of the distribution. When $\alpha_1 = \alpha_2 = \alpha_3 = 1$, the data points are uniformly distributed across the simplex, corresponding to a three-dimensional uniform distribution (as shown in 2(a)). When $\alpha_1 < 1$, $\alpha_2 < 1$, $\alpha_3 < 1$, the data points tend to cluster at the corners and edges of the simplex, creating generalized U-shaped distribution in three dimensions (see Figure 2(b)). When $\alpha_1 > 1$, $\alpha_2 > 1$, $\alpha_3 > 1$, the distribution exhibits a concentration of points at the simplex's center, forming a generalized bell-shaped pattern in three dimensions (see Figure 2 (c)). Figures 2 (d) - 2 (f) illustrate other shapes generated by different $(\alpha_1,\ \alpha_2,\ \alpha_3)$ combinations, each with a generalized J-shape in three dimensions.

We now derive the expected probability of labor supply from equation (5) under the assumption that the probabilities $(\pi_1,\ \pi_2,\ \pi_3)$ follow a Dirichlet distribution:

$$
\begin{aligned}
E\left[P\left(x_1,x_2,x_3,T\right)\right] &= \int \frac{T!}{x_1!x_2!x_3!}\pi_1^{x_1}\pi_2^{x_2}\left(\pi_3\right)^{x_3} \times \frac{1}{B(\alpha_1,\alpha_2,\alpha_3)}\pi_1^{\alpha_1-1}\pi_2^{\alpha_2-1}\pi_3^{\alpha_3-1}d\boldsymbol{\pi} \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{1}{B\left(\alpha_1,\alpha_2,\alpha_3\right)}\int \pi_1^{x_1+\alpha_1-1}\pi_2^{x_2+\alpha_2-1}\left(\pi_3\right)^{x_3+\alpha_3-1}d\boldsymbol{\pi}
\end{aligned}
\tag{7}
$$

Define

$$
\begin{aligned}
p &= Pr(y>2) = \pi_3 \\
q &= Pr(y>1) = \pi_2+\pi_3
\end{aligned}
\quad ,
$$

and

$$
\begin{aligned}
\pi_1 &= 1-q \\
\pi_2 &= q-p \\
\pi_3 &= p
\end{aligned}
\quad .
$$

Then

$$
E\left(P\left(x_1,x_2,x_3,T\right)\right) = \frac{T!}{x_1!x_2!x_3!}\frac{1}{B\left(\alpha_1,\alpha_2,\alpha_3\right)}\int_0^1\int_0^q (1-q)^{x_1+\alpha_1-1}(q-p)^{x_2+\alpha_2-1}p^{x_3+\alpha_3-1}dpdq
\tag{8}
$$

By the generalized binomial theorem:

$$
(q-p)^{x_2+\alpha_2-1} = \sum_{k=0}^{\infty}(-1)^k\begin{pmatrix} x_2+\alpha_2-1 \\ k \end{pmatrix} q^{x_2+\alpha_2-k-1}p^k
$$

Therefore

$$
\begin{aligned}
E\left(P\left(x_1,x_2,x_3,T\right)\right) &= \frac{T!}{x_1!x_2!x_3!}\frac{1}{B\left(\alpha_1,\alpha_2,\alpha_3\right)}\sum_{k=0}^{\infty}(-1)^k\begin{pmatrix} x_2+\alpha_2-1 \\ k \end{pmatrix} \\
&\times \int_0^1\int_0^q (1-q)^{x_1+\alpha_1-1}q^{x_2+\alpha_2-k-1}p^k p^{x_3+\alpha_3-1}dpdq \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{1}{B\left(\alpha_1,\alpha_2,\alpha_3\right)}\sum_{k=0}^{\infty}(-1)^k\begin{pmatrix} x_2+\alpha_2-1 \\ k \end{pmatrix} \\
&\times \int_0^1 (1-q)^{x_1+\alpha_1-1}q^{x_2+\alpha_2-k-1}\int_0^q p^{x_3+\alpha_3+k-1}dpdq \quad .
\end{aligned} \tag{9}
$$

The first integral in the last line above is as standard beta-logistic model, while the second integral is:

$$
\begin{aligned}
&\int_0^q p^{x_3+\alpha_3+k-1}dp \\
&= \left.\frac{p^{x_3+\alpha_3+k}}{x_3+\alpha_3+k}\right|_{p=0}^{p=q} = \frac{q^{x_3+\alpha_3+k}}{x_3+\alpha_3+k}
\end{aligned}
$$

So

$$
\begin{aligned}
E\left(P\left(x_1,x_2,x_3,T\right)\right) &= \frac{T!}{x_1!x_2!x_3!}\frac{1}{B\left(\alpha_1,\alpha_2,\alpha_3\right)}\sum_{k=0}^{\infty}(-1)^k\left(\begin{array}{c} x_2+\alpha_2-1 \\ k \end{array}\right) \\
&\times \frac{1}{x_3+\alpha_3+k}\int_0^1 (1-q)^{x_1+\alpha_1-1}q^{x_2+\alpha_2-k-1}q^{x_3+\alpha_3+k}dq \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{1}{B\left(\alpha_1,\alpha_2,\alpha_3\right)}\sum_{k=0}^{\infty}(-1)^k\left(\begin{array}{c} x_2+\alpha_2-1 \\ k \end{array}\right) \\
&\times \frac{1}{x_3+\alpha_3+k}\int_0^1 (1-q)^{x_1+\alpha_1-1}q^{x_2+x_3+\alpha_2+\alpha_3-1}dq \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{1}{B\left(\alpha_1,\alpha_2,\alpha_3\right)} \\
&\times \sum_{k=0}^{\infty}(-1)^k\left(\begin{array}{c} x_2+\alpha_2-1 \\ k \end{array}\right)\frac{B\left(x_1+\alpha_1,x_2+x_3+\alpha_2+\alpha_3\right)}{x_3+\alpha_3+k} \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{B\left(x_1+\alpha_1,x_2+x_3+\alpha_2+\alpha_3\right)}{B\left(\alpha_1,\alpha_2,\alpha_3\right)} \\
&\times \sum_{k=0}^{\infty}(-1)^k\frac{\Gamma(x_2+\alpha_2)}{\Gamma(k+1)\Gamma(x_2+\alpha_2-k)}\frac{1}{x_3+\alpha_3+k} \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{B\left(x_1+\alpha_1,x_2+x_3+\alpha_2+\alpha_3\right)}{B\left(\alpha_1,\alpha_2,\alpha_3\right)}\Gamma(x_2+\alpha_2) \\
&\times \sum_{k=0}^{\infty}(-1)^k\frac{1}{\Gamma(k+1)\Gamma(x_2+\alpha_2-k)(x_3+\alpha_3+k)} \quad .
\end{aligned} \tag{10}
$$

Note that

$$
\begin{aligned}
&\lim_{n\to\infty}\sum_{k=0}^{n}(-1)^k\frac{1}{\Gamma(k+1)\Gamma(x_2+\alpha_2-k)(x_3+\alpha_3+k)} \\
&= \lim_{n\to\infty}\frac{\frac{(-1)^n {}_3F_2(1,a+n+1,-b+n+2;n+2,a+n+2;1)}{\Gamma(n+2)\Gamma(b-n-1)}+\frac{n\Gamma(a+1)}{a\Gamma(a+b)}+\frac{(a+1)\Gamma(a+1)}{a\Gamma(a+b)}}{a+n+1} \quad ,
\end{aligned}
$$

where $a = x_3+\alpha_3$ and $b = x_2+\alpha_2$. Given that a and b are constants, as n goes to infinity, the third term of the limit goes to zero. We will now show the limit of the first term also equals zero. ${}_3F_2(1,a+n+1,-b+n+2;n+2,a+n+2;1)$ is a generalized hypergeometric function defined from the hypergeometric series, ${}_3F_2(1,a+n+1,-b+n+2;n+2,a+n+2;1) = \sum_{k=0}^{\infty} c_k$, for which $c_0 = 1$ and

$$\begin{aligned}\frac{c_{k+1}}{c_k} &= \frac{P(k)}{Q(k)} \\ &= \frac{(k+1)(k+a+n+1)(k-b+n+2)}{(k+n+2)(k+a+n+2)(k+1)} \\ &= \frac{(k+a+n+1)(k-b+n+2)}{(k+a+n+2)(k+n+2)} < 1 \quad .\end{aligned}$$

So,

$${}_3F_2(1, a+n+1, -b+n+2; n+2, a+n+2; 1) = \sum_{k=0}^{\infty} c_k < \sum_{k=0}^{\infty} c_0 = \sum_{k=0}^{\infty} 1 = \lim_{n\to\infty} \sum_{k=0}^{n} 1 = \lim_{n\to\infty} n$$

Hence,

$$\begin{aligned}&\lim_{n\to\infty} \frac{\frac{(-1)^n {}_3F_2(1,a+n+1,-b+n+2;n+2,a+n+2;1)}{\Gamma(n+2)\Gamma(b-n-1)}}{a+n+1} \\ &= \lim_{n\to\infty} \frac{\frac{(-1)^n \lim_{n\to\infty} n}{\Gamma(n+2)\Gamma(b-n-1)}}{a+n+1} \to 0\end{aligned}$$

So the limit of the first term also equals zero, and

$$\begin{aligned}&\lim_{n\to\infty} \frac{\frac{(-1)^n {}_3F_2(1,a+n+1,-b+n+2;n+2,a+n+2;1)}{\Gamma(n+2)\Gamma(b-n-1)} + \frac{n\Gamma(a+1)}{a\Gamma(a+b)} + \frac{(a+1)\Gamma(a+1)}{a\Gamma(a+b)}}{a+n+1} \\ &= \frac{\Gamma(a+1)}{a\Gamma(a+b)} = \frac{\Gamma(a)}{\Gamma(a+b)}\end{aligned}$$

Therefore,

$$\begin{aligned}E\left(P\left(x_1, x_2, x_3, T\right)\right) &= \frac{T!}{x_1!x_2!x_3!} \frac{B\left(x_1+\alpha_1, x_2+x_3+\alpha_2+\alpha_3\right)}{B\left(\alpha_1, \alpha_2, \alpha_3\right)} \frac{\Gamma(x_2+\alpha_2)\Gamma(x_3+\alpha_3)}{\Gamma(x_2+x_3+\alpha_2+\alpha_3)} \\ &= \frac{T!}{x_1!x_2!x_3!} \frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} \\ &\times \frac{\Gamma(x_1+\alpha_1)\Gamma(x_2+x_3+\alpha_2+\alpha_3)}{\Gamma(x_1+x_2+x_3+\alpha_1+\alpha_2+\alpha_3)} \frac{\Gamma(x_2+\alpha_2)\Gamma(x_3+\alpha_3)}{\Gamma(x_2+x_3+\alpha_2+\alpha_3)} \\ &= \frac{T!}{x_1!x_2!x_3!} \frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} \frac{\Gamma(x_1+\alpha_1)\Gamma(x_2+\alpha_2)\Gamma(x_3+\alpha_3)}{\Gamma(x_1+x_2+x_3+\alpha_1+\alpha_2+\alpha_3)}\end{aligned} \tag{11}$$

The model is complted by adding individual heterogeneity:

$$\begin{aligned}
\alpha_1 &= \frac{exp(z'\beta_1)exp(z'\beta_3) + exp(z'\beta_1)exp(z'\beta_4)}{exp(z'\beta_1) + exp(z'\beta_2)} \\
\alpha_2 &= \frac{exp(z'\beta_2)exp(z'\beta_3) - exp(z'\beta_1)exp(z'\beta_4)}{exp(z'\beta_1) + exp(z'\beta_2)} \\
\alpha_3 &= exp(z'\beta_4)
\end{aligned} .$$

Then for $T = 1$:

$$E\left(P\left(1,0,0,1\right)\right) = \frac{exp(z'\beta_1)}{exp(z'\beta_1) + exp(z'\beta_2)}$$

$$E\left(P\left(0,1,0,1\right)\right) = \frac{exp(z'\beta_2)}{exp(z'\beta_1) + exp(z'\beta_2)} - \frac{exp(z'\beta_4)}{exp(z'\beta_3) + exp(z'\beta_4)}$$

$$E\left(P\left(0,0,1,1\right)\right) = \frac{exp(z'\beta_4)}{exp(z'\beta_3) + exp(z'\beta_4)}$$

with respective variances:

$$\begin{aligned}
\sigma_1^2 &= E[(p(1,0,0,1))^2] - E[(p(1,0,0,1))]^2 \\
&= \frac{\alpha_1(\alpha_2 + \alpha_3)}{(\alpha_1 + \alpha_2 + \alpha_3)^2(1 + \alpha_1 + \alpha_2 + \alpha_3)}
\end{aligned}$$

$$\begin{aligned}
\sigma_2^2 &= E[(p(0,1,0,1))^2] - E[(p(0,1,0,1))]^2 \\
&= \frac{\alpha_2(\alpha_1 + \alpha_3)}{(\alpha_1 + \alpha_2 + \alpha_3)^2(1 + \alpha_1 + \alpha_2 + \alpha_3)}
\end{aligned}$$

$$\begin{aligned}
\sigma_3^2 &= E[(p(0,0,1,1))^2] - E[(p(0,0,1,1))]^2 \\
&= \frac{\alpha_3(\alpha_1 + \alpha_2)}{(\alpha_1 + \alpha_2 + \alpha_3)^2(1 + \alpha_1 + \alpha_2 + \alpha_3)}
\end{aligned}$$

This defines the Dirichlet generalized ordered logit model. With cross-section data (i.e., data on participation for only one year), β_1 and β_2 (β_3 and β_4) cannot be identified separately. Hence, the traditional generolized ordered logit function can be used only to predict the mean participation rate ($\beta_2 - \beta_1$) and $\beta_4 - \beta_3$) in a population conditional on the z's, but cannot determine the higher moments of the distribution of participation probabilities. However, with participation data on the same individuals for 2 or more years, both β_1 and β_2 (β_3 and β_4) can be identified.

The spurious state dependence induced by heterogeneity may be seen by comparing the conditional probability of full time working in year t of women who worked full time in year $t-1$ with that of women who did not work in $t-1$ or worked part time in $t-1$:

$$
\begin{aligned}
&P(y_t = 3|y^*_{t-1} = 3) - P(y_t = 3|y^*_{t-1} = 1)\\
&= \frac{\alpha_3 + 1}{1 + \alpha_1 + \alpha_2 + \alpha_3} - \frac{\alpha_3}{1 + \alpha_1 + \alpha_2 + \alpha_3}\\
&= \frac{1}{1 + \alpha_1 + \alpha_2 + \alpha_3}\\
&= \frac{exp(z'\beta_1) + exp(z'\beta_2)}{exp(z'\beta_1) + exp(z'\beta_2) + exp(z'\beta_1 + z'\beta_3) + exp(z'\beta_1 + z'\beta_4) + exp(z'\beta_2 + z'\beta_3) + exp(z'\beta_2 + z'\beta_4)}\ .
\end{aligned}
$$

and

$$P(y_t = 3|y^*_{t-1} = 3) - P(y_t = 3|y^*_{t-1} = 2) = \frac{1}{1 + \alpha_1 + \alpha_2 + \alpha_3}$$

This difference ranges from zero (i.e. $\sigma^2 \to 0$ as $\alpha_1, \alpha_2, \alpha_3 \to \infty$) to one under extreme heterogeneity (i.e. $\sigma^2 \to \infty$ as $\alpha_1, \alpha_2, \alpha_3 \to 0$).

The conditional probability of working full-time (given previous full-time work) rises. Thus,

$$
\begin{aligned}
&P(y_t = 3|y_{t-1} = 3, \ldots, y_1 = 3)\\
&= \frac{P(y_t = 3, y_{t-1} = 3, \ldots, y_1 = 3)}{P(y_{t-1} = 3, \ldots, y_1 = 3)}\\
&= \frac{\alpha_1 + t - 1}{\alpha_1 + \alpha_2 + \alpha_3 + t - 1}\\
&= \left\{exp(z'\beta_1 + z'\beta_3) + exp(z'\beta_1 + z'\beta_4) + (exp(z'\beta_1) + exp(z'\beta_2))(t-1)\right\} \Big/\\
&\quad \Big\{exp(z'\beta_1 + z'\beta_3) + exp(z'\beta_1 + z'\beta_4) + exp(z'\beta_2 + z'\beta_3) + exp(z'\beta_2 + z'\beta_4)\\
&\quad + (exp(z'\beta_1) + exp(z'\beta_2))(t-1)\Big\}\ ,
\end{aligned}
$$

which is a positive monotonic function of t that approaches unity as t approaches infinity. Ssimilar equation can be derived for the conditional probabilies of not working and working part-time.

The predictive probability of the labor force participation decision at time t, based on the previous pattern of job participation behavior up to time $t-1$, can be expressed as:

$$
\begin{aligned}
f_{1t} &= P(y_t = 1 | x_{1,t-1}, x_{2,t-1}, x_{3,t-1}, t-1) \\
&= \frac{E[p(x_{1,t-1}+1, x_{2,t-1}, x_{3,t-1}, t)]}{E[p(x_{1,t-1}, x_{2,t-1}, x_{3,t-1}, t-1)]} \\
&= \frac{x_{1,t-1}+\alpha_1}{t-1+\alpha_1+\alpha_2+\alpha_3} \\
&= \Big\{(exp(z'\beta_1)+exp(z'\beta_2))x_{1,t-1}+exp(z'\beta_1+z'\beta_3)+exp(z'\beta_1+z'\beta_4)\Big\} \Big/ \\
&\quad \Big\{(exp(z'\beta_1)+exp(z'\beta_2))(t-1)+exp(z'\beta_1+z'\beta_3) \\
&\quad +exp(z'\beta_1+z'\beta_4)+exp(z'\beta_2+z'\beta_3)+exp(z'\beta_2+z'\beta_4)\Big\} \\
f_{2t} &= P(y_t = 2 | x_{1,t-1}, x_{2,t-1}, x_{3,t-1}, t-1) \\
&= \frac{E[p(x_{1,t-1}, x_{2,t-1}+1, x_{3,t-1}, t)]}{E[p(x_{1,t-1}, x_{2,t-1}, x_{3,t-1}, t-1)]} \\
&= \frac{x_{2,t-1}+\alpha_2}{t-1+\alpha_1+\alpha_2+\alpha_3} \\
&= \Big\{(exp(z'\beta_1)+exp(z'\beta_2))x_{2,t-1}+exp(z'\beta_2+z'\beta_3)-exp(z'\beta_1+z'\beta_4)\Big\} \Big/ \\
&\quad \Big\{(exp(z'\beta_1)+exp(z'\beta_2))(t-1)+exp(z'\beta_1+z'\beta_3) \\
&\quad +exp(z'\beta_1+z'\beta_4)+exp(z'\beta_2+z'\beta_3)+exp(z'\beta_2+z'\beta_4)\Big\} \\
f_{3t} &= P(y_t = 3 | x_{1,t-1}, x_{2,t-1}, x_{3,t-1}, t-1) \\
&= \frac{E[p(x_{1,t-1}, x_{2,t-1}, x_{3,t-1}+1, t)]}{E[p(x_{1,t-1}, x_{2,t-1}, x_{3,t-1}, t-1)]} \\
&= \frac{x_{3,t-1}+\alpha_3}{t-1+\alpha_1+\alpha_2+\alpha_3} \\
&= \Big\{(exp(z'\beta_1)+exp(z'\beta_2))x_{3,t-1}+exp(z'\beta_1)^2+exp(z'\beta_1+z'\beta_2)\Big\} \Big/ \\
&\quad \Big\{(exp(z'\beta_1)+exp(z'\beta_2))(t-1)+exp(z'\beta_1+z'\beta_3) \\
&\quad +exp(z'\beta_1+z'\beta_4)+exp(z'\beta_2+z'\beta_3)+exp(z'\beta_2+z'\beta_4)\Big\} \;,
\end{aligned}
\tag{12}
$$

where y_t represents the labor force participation decision at time t. y_t equals 1 for not working at time t, y_t equals 2 for working part-time at time t and y_t equals 3 for working full-time at time t. $x_{i,t-1}$ is the cumulative number of times that choice i has been selected in the preceding $t-1$ periods, where $\sum x_{i,t-1} = t-1$. These equations demonstrate that the predictive probabilities of women's labor force participation behavior at time t depends not only on exogenous factors but also on their past behavior.

3 Data

3.1 Data Description

This research uses data from waves 1 through 19 (1997-2019) of the NLSY97 cohort. The NLSY97, a part of the US Department of Labor's survey series, explores workforce entry patterns and transitions for youth. The content of the survey includes extensive information about youths' employment history, educational experiences, marital status, etc. over time. It is a nationally representative panel study tracking individuals aged 12 to 16 on December 31, 1996. The initial 1997 survey covered 8,984 respondents, with 51 percent men (4,599) and 49 percent women (4,385).

We analyze NLSY97 women who gave birth to their first child and had employment status data for the five years following their childbirth, starting from the first year after. Similar to Deza (2023), we categorize employment into three levels based on employment status and work hours: non-working, part-time worker, and full-time worker. The NLSY97 provides a comprehensive employer roster for each respondent at every wave, detailing all employers they worked for since the last interview. This roster differentiates between types of jobs, such as employee roles, freelance positions, self-employment, or military service. Using information on whether the respondent is matched to at least one employer and the number of work hours, We determine whether the respondent worked since the last interview and compute the total number of hours. We define not working as reporting zero hours of work in this year. Part-time workers are defined as those who worked at most 1500 hours a year. Finally, full-time workers are those that report working more than 1500 hours in this year.

Table 1 presents the definitions of the main variables. The explanatory variables consist of education in years at childbirth, age one year after childbirth, having a spouse or partner at childbirth, logarithm of the partner's annual real income, logarithm of the mother's hourly real wage for the job at childbirth or immediately preceding it, religion indicators, and indicators for US region. Additionally, two extra variables are included. The first one comprises missing value indicators for the logarithm of the spouse or partner's real income, while the second pertains to missing values for the logarithm of the mother's real wage. The estimated coefficients for these two variables are not presented in the regression table.

Table 2 displays the summary statistics: 21.5 percent of women choose not to work after childbirth, while 43.8 persent work full-time and 34.8 percent work part-time. The average age at the birth of their first child is 24 years. Figure 3 indicates that most women give birth between the ages of 20 to 30. The average education level exceeds high school, with women having an average of 12.4 years of education. Figure 4 presents a detailed distribution of different education levels. Additionally, 63.1 percent of women have a partner at the time of childbirth.

3.2 Pre-sample Analysis

Figure 5 shows the number of weeks women take to return to work, truncated at 104 weeks. The data shows that 25 percent of women return within four weeks, 50 percent within three months, and 75 percent within around a year. This distribution further justifies our choice to commence our study from one year post-childbirth. Because women may experience different health conditions and job search capabilities immediately after childbirth, a one-year window allows for the accommodation of individual differences during this transition period. Additionally, the distribution indicates that about 75 percent of women resume work within a year, which further confirms the suitability of this one-year window for our selection.

We use a nested logit model to estimate the probability of women returning to employment and returning their previous employer in the year after childbirth. We include characteristics and three other variables in the model: 'education of the woman's mother', 'work duration' and 'percentage of time in the labor force'. The nested logit model is displayed in Figure 6. Initially, we categorize employment decisions into two primary groups: choosing not to work and deciding to return to work. For those in the second category, we further distinguish between two scenarios: returning to the same employer and returning to work with a new employer.

The nested logit results are presented in Table 3. Women with higher education and wages are more likely to work and return to their previous employers, while older women tend to not work or seek a different employer. Having a partner increases the likelihood of returning to work and to a previous employer. Women work less when the income of their partner is higher. The education level of a woman's mother does not directly influence her work decisions. A longer duration of work before childbirth reduces the probability of returning to work within a year after childbirth. Conversely, a greater percentage of time working before childbirth increases the likelihood of returning to work and to the previous employer. Figures 7 and 8 show the distribution of the probability of returning to work and returning to the same employer within one year.We will use these two probabilities into our secend stage ordered logit model.

4 Empirical Results

4.1 Parameter Estimates

We first run the ordered logit model, generalized ordered logit model, and our DGOL model incorporating only two probabilities. Tables 4 and 5 present the parameter estimates derived from these regressions. The coefficients in these tables reveal that a higher probability of returning to work and returning to the same employer within a year after childbirth increases the likelihood of women working and working full-time in the subsequent years.

In Tables 6 through 8, we extend our analysis by including women's characteristics variables. Our focus is on comparing the results of the generalized ordered logit model (Table 7) with those of the DGOL model (Table 8). Analyzing DGOL coefficients can be somewhat intricate, as it involves four columns. In the case of the generalized ordered logit model, the baselines are set to zero for each comparison. For example, in the comparison of working versus not working, 'not working' is the baseline, and in the comparison of full-time versus not full-time, 'not full-time' is the baseline. However, in the DGOL when period T greater than 1, we have values for the baseline choice. In the next subsection, we will delve into analyzing the average partial effects (APEs) to gain further insights into state dependence.

The signs of most variables are consistent across models and align with our expectations. Similar to the results for previous models, a higher probability of returning to work and returning to the same employer within a year increases the likelihood of women working and reduces the likelihood of not working. Furthermore, women with higher education are more likely to work, specifically in full-time positions, and less likely to remain non-working. On the other hand, older women are less likely to return to work after childbirth.

Generalized ordered logit estimation shows that partnership, partner income, and hourly wage have negligible impacts on women's employment decisions. This finding is counter intuitive, as numerous studies emphasize the importance of wage levels and partnership status on women's labor force participation (Hausman 1979; Treas 1987; Bradbury and Katz 2002; Bowen and Finegan 2015). In contrast, the DGOL model reveals that partnership and higher partner income are associated with a decreased likelihood of women working after childbirth. Moreover, wage levels are a significant factor motivating women to work after childbirth. Finally, region has minimal impact on women's labor force participation decisions in the generalized ordered logit model, while is significant in the DGOL.

4.2 Model Accuracy

We present the accuracy rates for three models in Figure 9. We begin by comparing the performance of the ordered logit and generalized ordered logit models. The generalized ordered logit outperforms the ordered logit slightly. In the case of the DGOL, the accuracy rate in the first year is comparable to that of the generalized ordered logit model. As discussed in Section 2, for cross-sectional data (only one year data), the Dirichlet generalized ordered logit degenerates to the generalized ordered logit. Therefore, their accuracy rates are nearly identical in the first year. However, in the second year, the Dirichlet generalized ordered logit, which considers heterogeneity, exhibits significant predictive improvement with an accuracy rate of 0.83, substantially surpassing the accuracy rates of 0.55 for the generalized ordered logit and 0.53 for the ordered logit models. This elevated accuracy persists in subsequent years.

Given that the dependent variable is ordinal, we compute the mean absolute error (MAE) for all three models[2]. Because accuracy rate merely indicates whether the predicted category is correct or not, without considering how far off a wrong prediction is. But MAE takes into account the order of the categories and the magnitude of the errors. A lower MAE value suggests superior predictive accuracy for a model. As depicted in Figure 10, all three models exhibit similar MAE values in the first year. However, in the second year, the DGOL's MAE drops from 0.50 to 0.18, notably outperforming the generalized ordered logit model's 0.53 and the ordered logit model's 0.55. The MAE values for DGOL remains small in the following years.

4.3 Heterogeneity

Figure 11 suggests the presence of considerable heterogeneity in the response probability. In a homogeneous population, the response probability distribution should be centered around

[2] The MAE formula is: MAE = $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$, where y_i is the actual dependent variable value, and $\hat{y}_i$ is the predicted dependent variable value. In our study, y_i and $\hat{y}_i$ represent the actual and predictive employment status.

the simplex's midpoint. However, the figure reveals an asymmetrical Dirichlet distribution concentrated tightly on one side of the simplex. This skewed distribution indicates that only a few women are likely to consider switching their employment status near the response mean. For most women, there seems to be a strong inclination and preference towards a specific employment status, resulting in this distribution. The mean response probability, estimated by the (generalized) ordered logit model, fails to adequately capture the nuances of women's labor force participation decisions. This is due to the model's assumption of uniformity within the women's population, which overlooks the diversity of experiences and factors influencing each individual's choice.

Next, we compute the average partial effects (APEs) using the parameter estimates to quantify the change in predicted probability of full-time employment that appears to be driven by prior full-time employment relative to prior non-full-time employment, holding other condition no change: $P(y_t = \text{full time}|y_{t-1} = \text{full time}) - P(y_t = \text{full time}|y_{t-1} = \text{not full time})$. That is, the causal effect of lagged employment action on current full-time employment decision. Given that individuals are only observed in one possible scenario ($\{y_{t-1} = \text{full time}\}$ or $\{y_{t-1} = \text{not full time}\}$), observed data would only allow us to compute either $P(y_t = \text{full time}|y_{t-1} = \text{full time})$ or $P(y_t = \text{full time}|y_{t-1} = \text{not full time})$. A naive method to get the APEs is by the difference of these two probabilities.

However, the group of people who worked full-time in time $t-1$ may be different from the group of people who didn't work full-time in time $t-1$, so the difference we get will be biased and this selection bias may mask the real APEs (Angrist and Pischke 2009). We simulate 100,000 artificial observations, ensuring that every variable has a distribution similar to that of the observed data. We randomly assign their employment status in the first period based on the proportion of each employment type in the observed data. Then, we use the estimated parameters from our structural DGOL model and equation (12) to predict their choice in subsequent years. Next, we estimate this difference in probabilities separately for each respondent and period. Finally, we take the average over all individuals and periods after the initial period to compute the APE.

The APEs from the DGOL model, as presented in Table 9, can be summarized as follows. The estimates indicate that if all respondents were to be artificially employed full-time in any given period while holding their characteristics unchanged, their probability of full-time employment in the next period would increase by 64.8 percentage points relative to the counterfactual scenario where they would be artificially not employed full-time in any given period (i.e., $P(y_t = \text{full time}|y_{t-1} = \text{full time}) - P(y_t = \text{full time}|y_{t-1} = \text{not full time}) = 0.648$). Given the existing literature on state dependence in employment (e.g. Deza 2023), it is unsurprising that $P(y_t = \text{full time})$ more than quadruples when we artificially change the lagged employment outcome from not-full-time to full-time employment (increasing from 0.191 to 0.839).

4.4 Policy Consequences of Job Continuity

In this subsection, we simulate full-time and non-employment under several different simulated policy counterfactual scenarios. Figures 12 and 13 indicates that the DGOL model closely simulates full-time employment patterns over the years. We assess full-time rates by artificially adjusting partner income levels to high, low, and no income scenarios. As shown in Figure 14, women with partners who have low or no income are more motivated to work full-time. The simulated full-time employment rate increases by approximately 5 percent for women with no partner income.

Using the parameters estimated by the DGOL model, we simulate full-time and non-employment rates after artificially adjusting the probability of returning to the same employer within a year after childbirth to high, moderate, and low levels. Figures 15 and 16 present the changes in (non-)employment rates in each scenario. Surprisingly, artificially increasing the probability of returning to the same employer to a high level significantly boosts the full-time employment rate, increasing it from 0.44 to 0.55. Conversely, the low probability of returning to the previous employer reduces the artificial full-time rate from 0.44 to 0.35. In terms of the non-employment rate in Figure 16, both low and moderate probabilities of returning to the same employment increase the non-employment rate by approximately 1 percent and 3 percent, while a high probability of returning to the same employment decreases the non-employment rate by around 3.5 percent.

Figure 17 depicts the simulated impact of job continuity-related policies on women with lower educational attainment. The green dotted line represents women with less than a high school education, showing that their full-time employment rate is lower than that of the entire population. However, if we increase the probability of returning to the same employment to a high level, this policy can mitigate the negative effect of low education on their full-time employment. The red dashed line demonstrates that the full-time employment rate will be slightly higher than the rate in the original simulated data.

Figure 18 illustrates the simulated impact of job continuity-related policies on older new mothers. The green dotted line highlights the labor market disadvantage faced by older new mothers, with a full-time employment rate approximately 5 percent below that of the entire population. However, the red dashed line demonstrates that implementing a job continuity policy, which increases their probability of returning to the same employer, can offset this disadvantage and result in a full-time employment rate similar to that of the entire population.

These policy counterfactuals hold significant value and offer insights into potential negative or positive outcomes related to existing job continuity policies, such as maternity leave and paid family leave. Our findings emphasize the crucial role of job continuity and indicate the necessity for job protection during the childbirth phase. Returning to one's previous employer after childbirth exerts a significant influence on women's subsequent employment choices. Policies aimed at promoting job continuity can substantially boost labor supply post-childbirth.

5 Conclusion

This study have a significant step forward in understanding the job continuity and other determinants of women's labor force participation in the United States. Considering the notable decline in women's labor supply following childbirth and the important role employment plays in economic prosperity, it is imperative for policymakers to grasp the interplay between job continuity and women's labor market success. The innovative approach of integrating unobserved heterogeneity and state dependence within the context of a higher-dimensional Dirichlet generalized ordered logit model offers a novel perspective on the complex factors influencing women's work choices after childbirth. The predictive accuracy achieved by our Dirichlet generalized ordered logit model, surpassing other established models.

The empirical findings of this study provide a deeper insight into the post-childbirth employment landscape for women, revealing substantial heterogeneity and state dependence within this population. Given the lack of natural experiments that randomly determine past employment statuses, prior research has struggled to discern the causal impact of previous employment behaviors on subsequent employment. Our findings indicate that, relative to other employment statuses, being engaged in full-time work during any period enhances the likelihood of maintaining in full-time employment in next year by 64.8 percent.

Moreover, our newly-developed model enables us to employ estimated parameters to simulate policy counterfactuals. These simulations suggest that when women are assigned a high probability of returning to the same employer post-childbirth, the rate of full-time employment surges by over 10 percent, with all else held unchanged. Interestingly, this heightened likelihood can also counterbalance the adverse effects of older age and lower education on women's post-childbirth full-time employment.

Our research illuminates the potential impacts of regulations aiming to bolster women's job continuity after parental leave on their labor market attachments in the United States. in the absence of policies or experiments targeting varying return-to-work likelihood, our policy counterfactuals become especially valuable. They highlight the potential necessary externalities of currently existing policies promoting women's re-entry into the labor market and safeguarding them from dismissal during maternity, such as maternity leave, paid family leave, and related job protection legislation.

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Figures

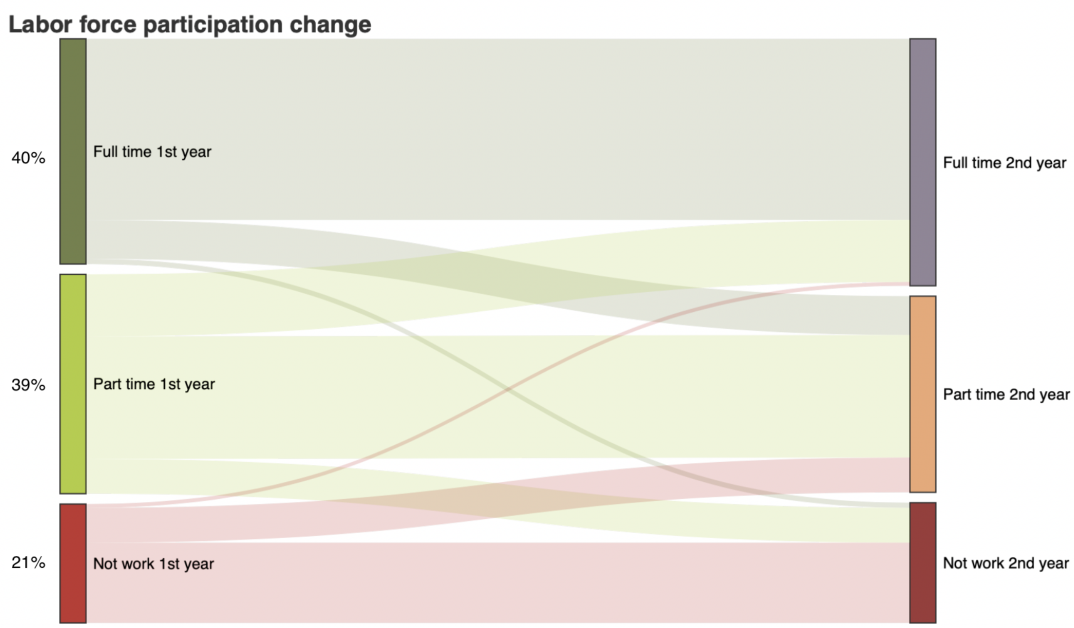

Figure 1: Women's Labor Force Participation: First to Second Year Transitions

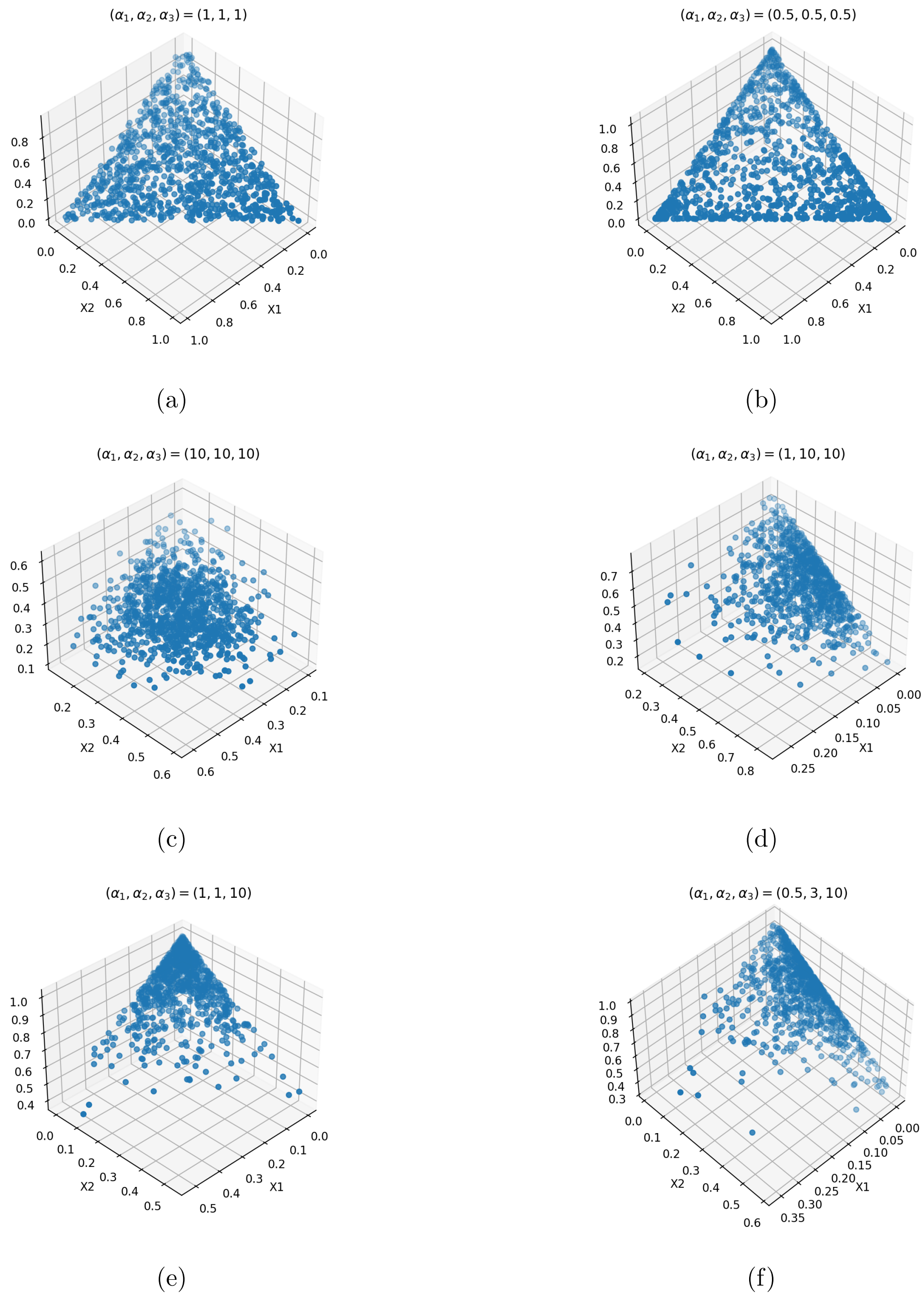

Figure 2: Scatter Plots of the Dirichlet Distribution

Notes: This figure shows the Dirichlet distributions under different shape parameters (α_1 α_2 α_3) for K = 3. Those shape parameters govern the shape of the distribution.

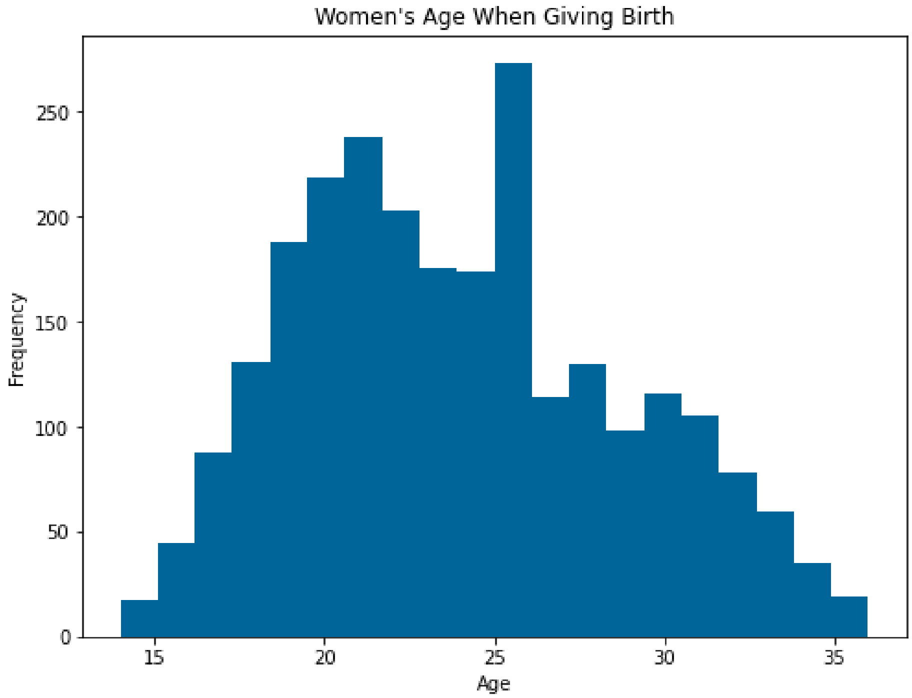

Figure 3: Women's Age When Giving Birth

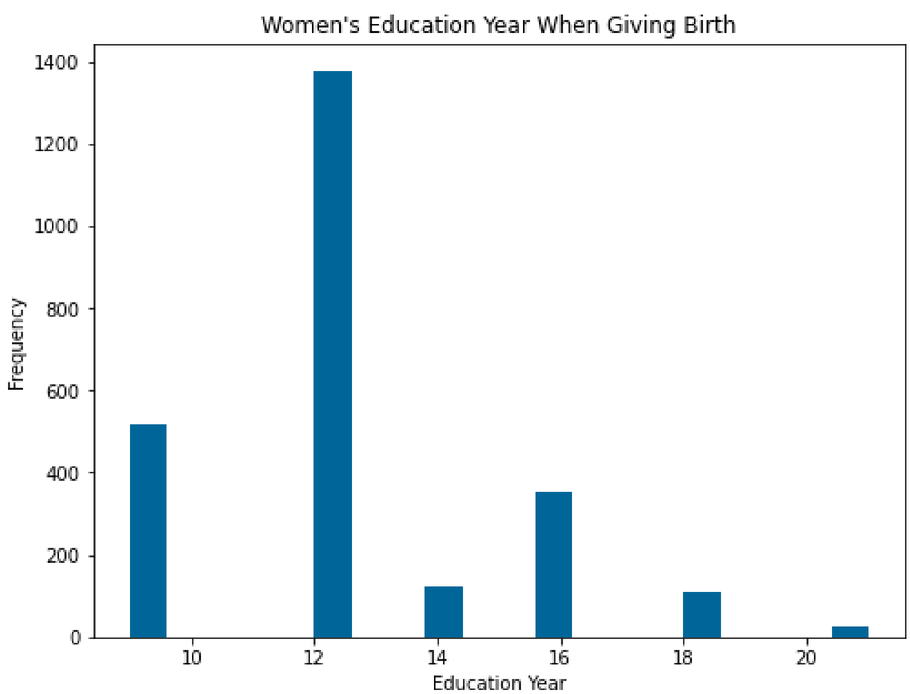

Figure 4: Women's Education Level (year) When Giving Birth

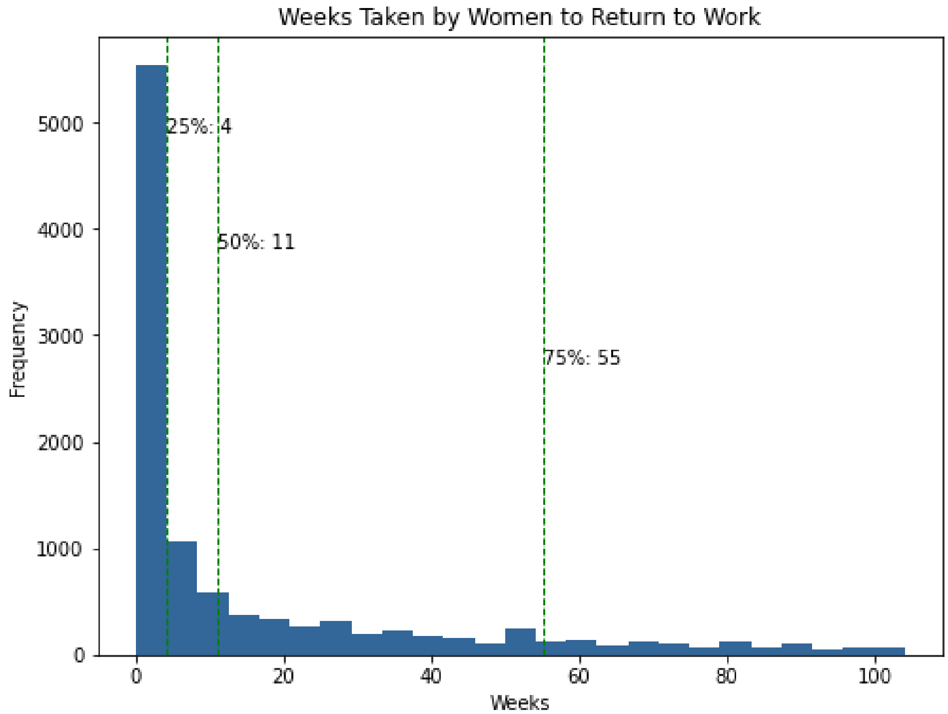

Figure 5: Weeks Taken by Women to Return to Work

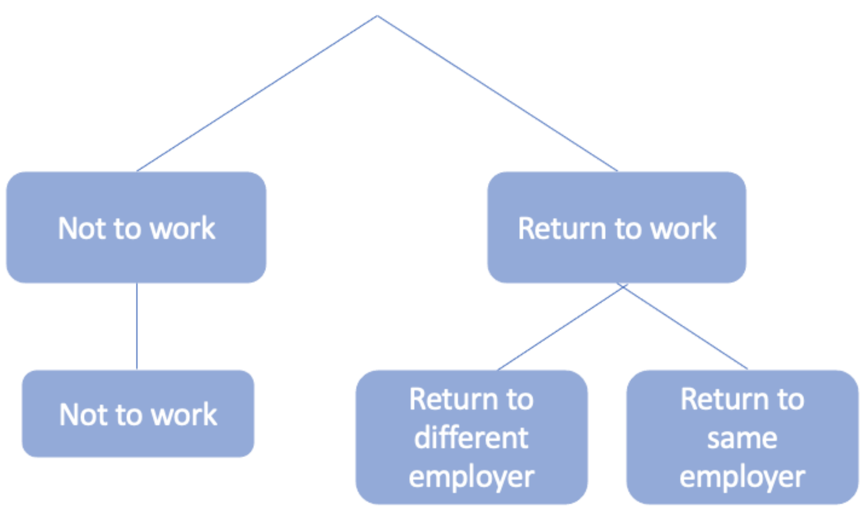

Figure 6: Nesting Structure for Pre-sample Employment

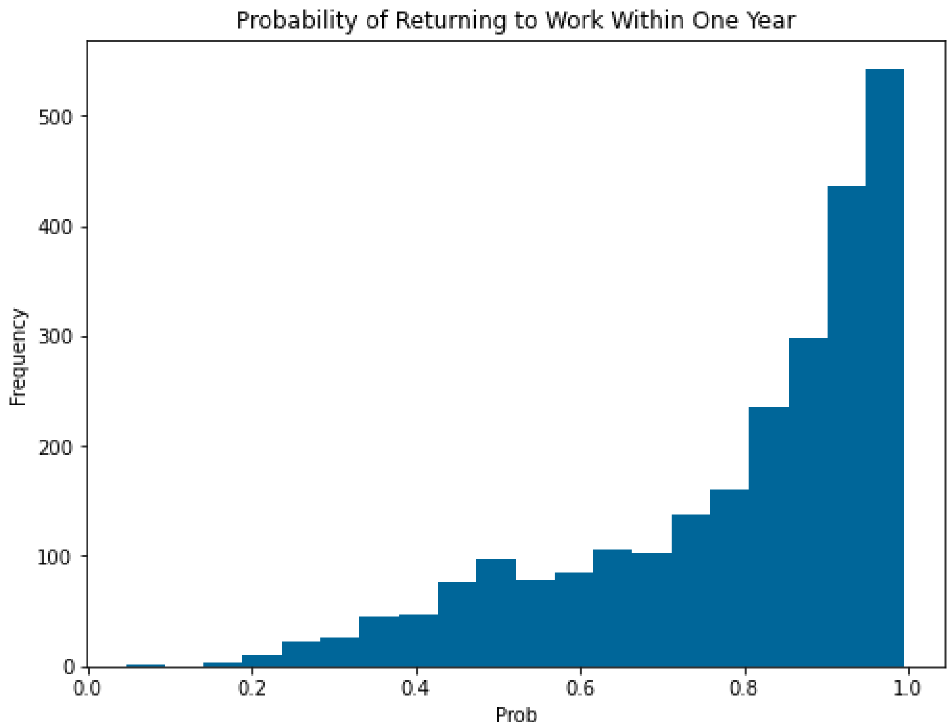

Figure 7: Probability of Returning to Work within One Year

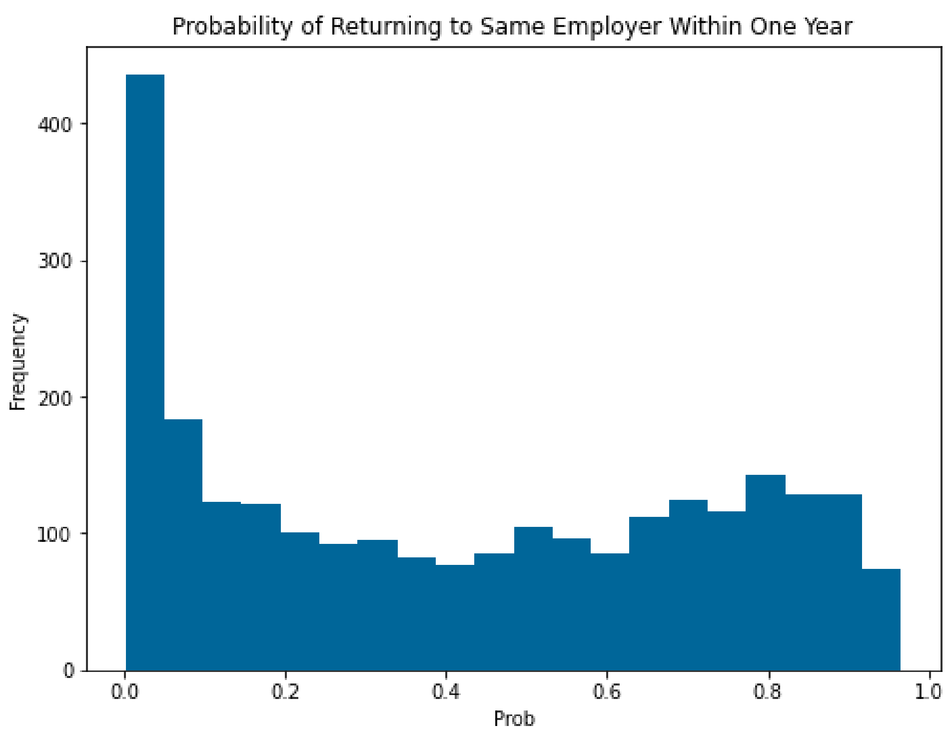

Figure 8: Probability of Returning to Same Employer within One Year

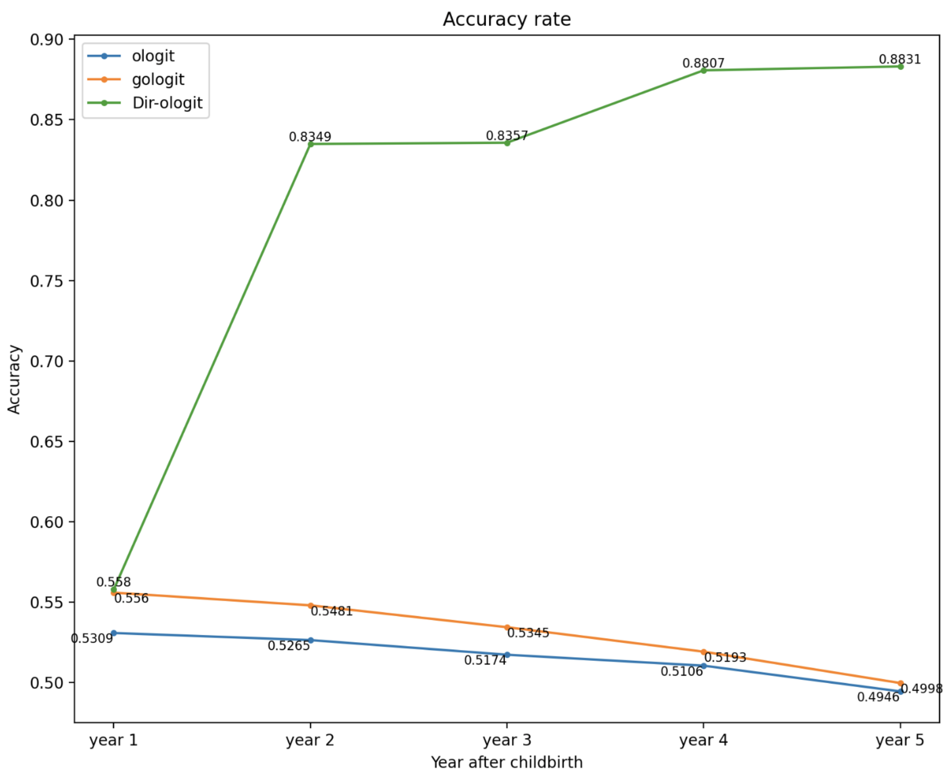

Figure 9: Model Comparison: Accuracy Rate

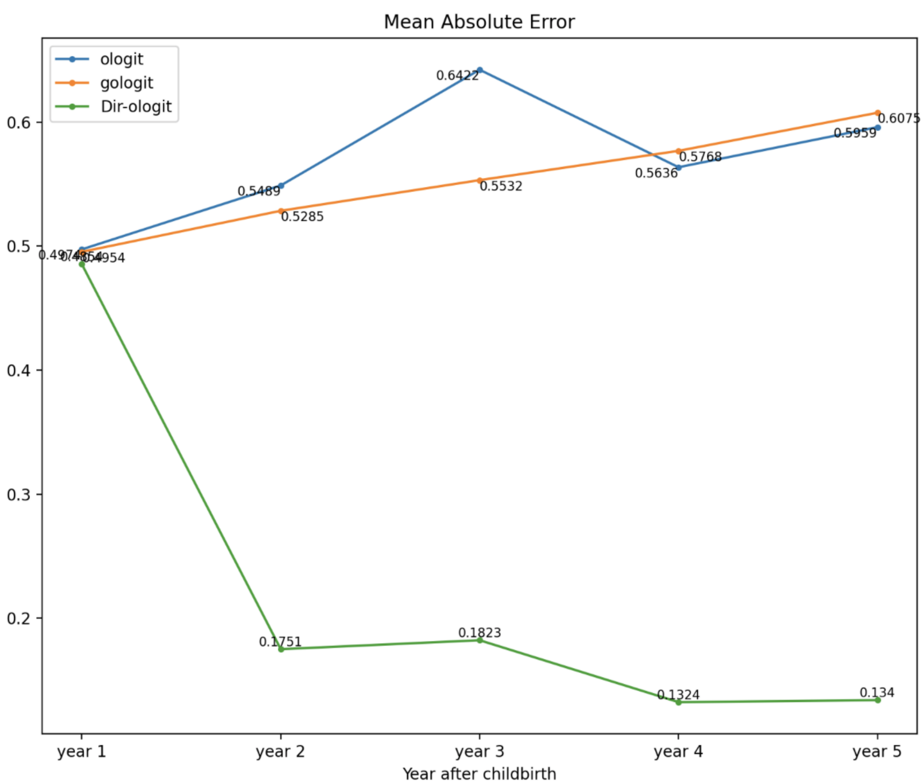

Figure 10: Model Comparison: Mean Absolute Error

Notes: The formula for MAE is given by: MAE = $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$, where y_i is the actual value, and $\hat{y}_i$ is the predicted value.

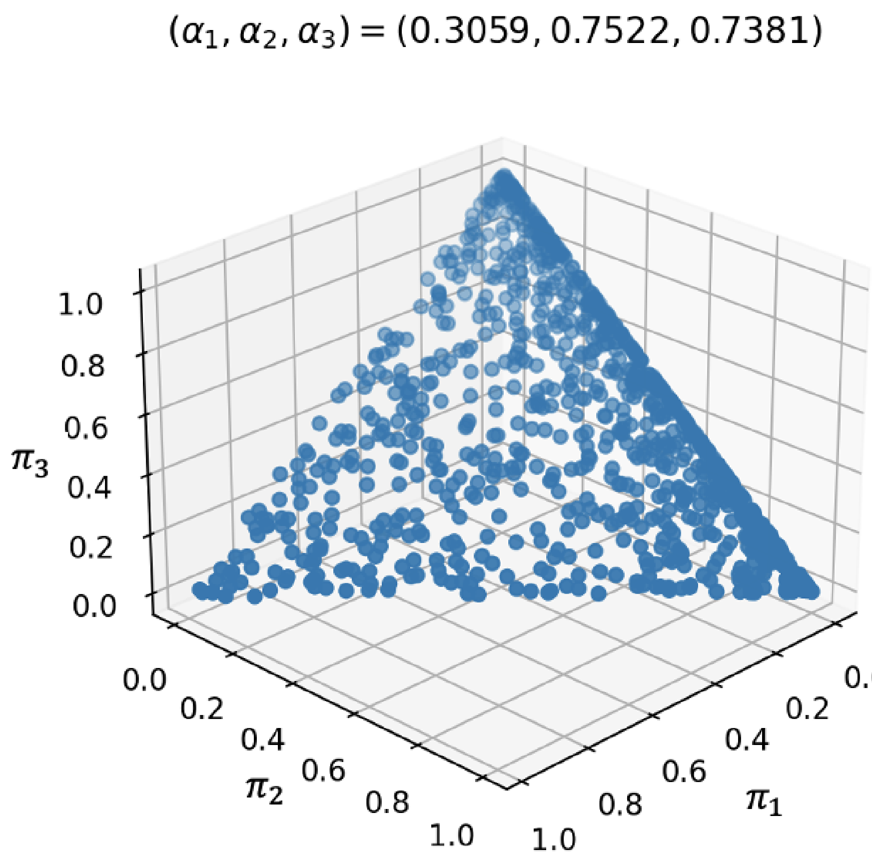

Figure 11: The Dirichlet Distribution for the DGOL Model 2

Note: This figure shows the Dirichlet distribution of full population with average characteristics for women. The definition of shape parameters $(\alpha_1, \alpha_2, \alpha_3)$ in the Dirichlet distribution are from Section 2.2. $\alpha_1 = \frac{exp(z'\beta_1)exp(z'\beta_3)+exp(z'\beta_1)exp(z'\beta_4)}{exp(z'\beta_1)+exp(z'\beta_2)}$, $\alpha_2 = \frac{exp(z'\beta_2)exp(z'\beta_3)-exp(z'\beta_1)exp(z'\beta_4)}{exp(z'\beta_1)+exp(z'\beta_2)}$ and $\alpha_3 = exp(z'\beta_4)$.

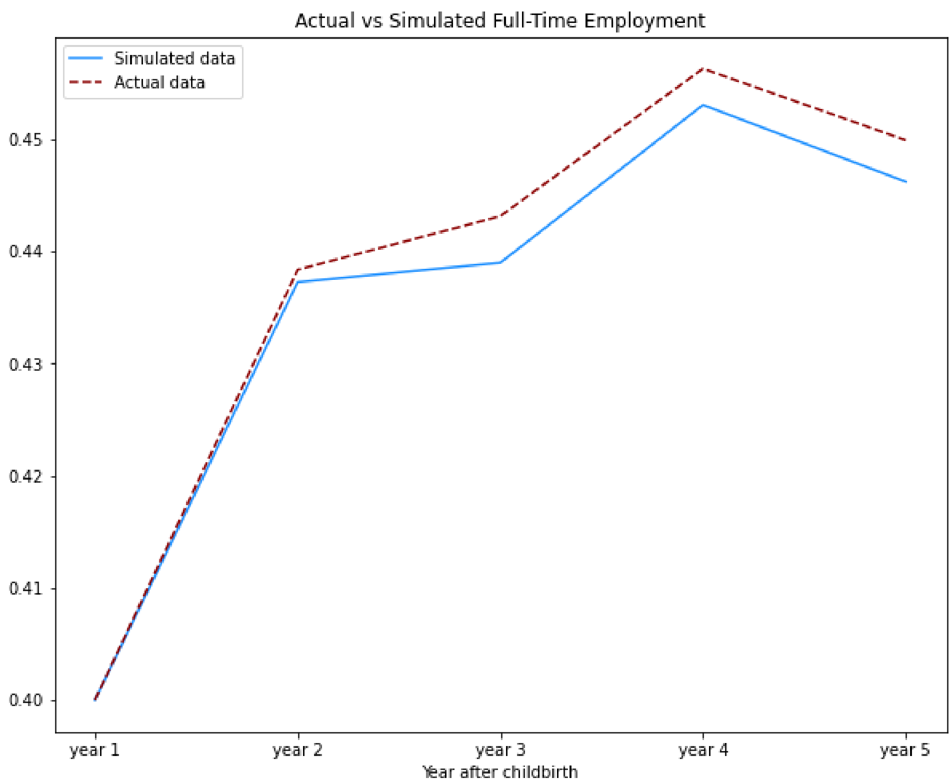

Figure 12: Actual vs Simulated Full-Time Employment

Notes: This figure presents the observed and simulated probability of full-time employment using the parameters estimated by the Dirichlete generalized ordered logit model.

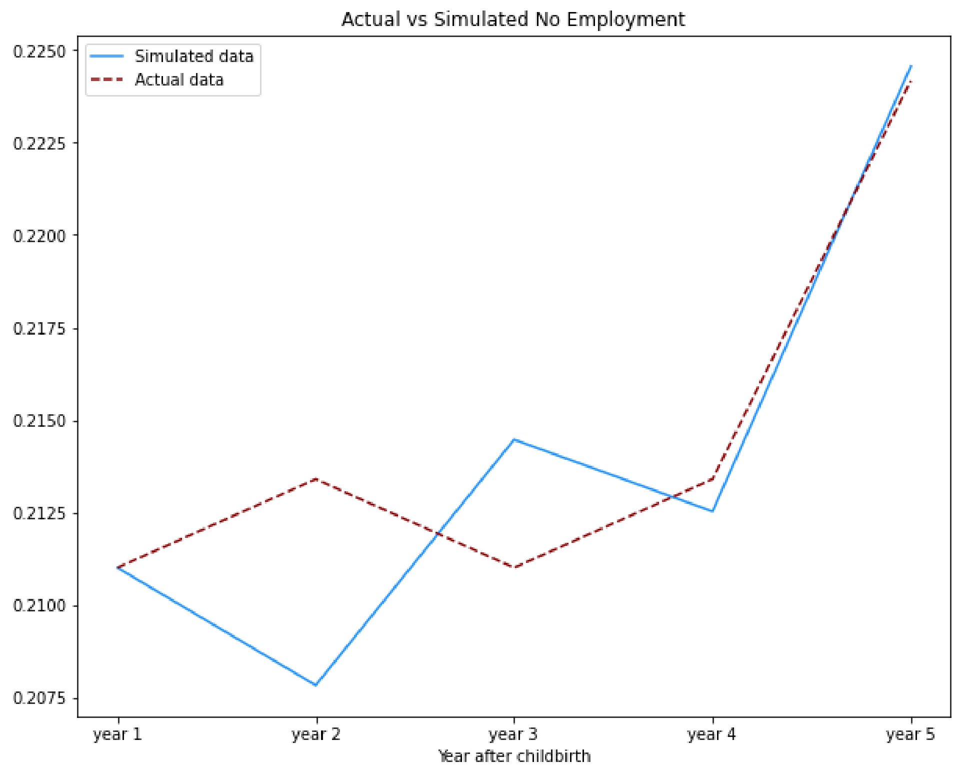

Figure 13: Actual vs Simulated Non-Employment

Notes: This figure presents the observed and simulated probability of non-employment using the parameters estimated by the Dirichlete generalized ordered logit model.

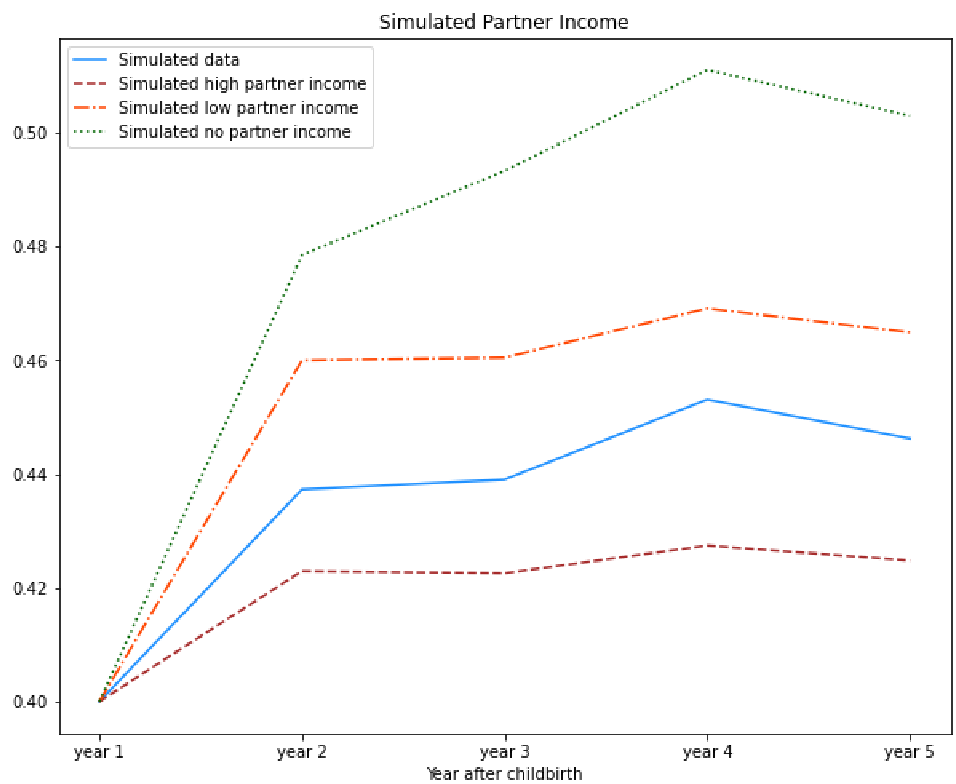

Figure 14: Simulated Partner Income impact (full time)

Notes: This figure presents the simulated probability of full-time employment using the parameters estimated by the Dirichlet generalized ordered logit model. In addition, it shows the simulated probability of full-time employment under three counterfactual scenarios, where I hold all other variables unchanged and artificially change the partner income for all respondents who have partner before simulating full-time employment in the next period: (i) A scenario with a high partner income (log (partner income) = 8) given they have partner, (ii) a scenario with partner income (log (partner income) = 5) given they have partner, and (iii) a scenario with no partner income (log (partner income) = 0).

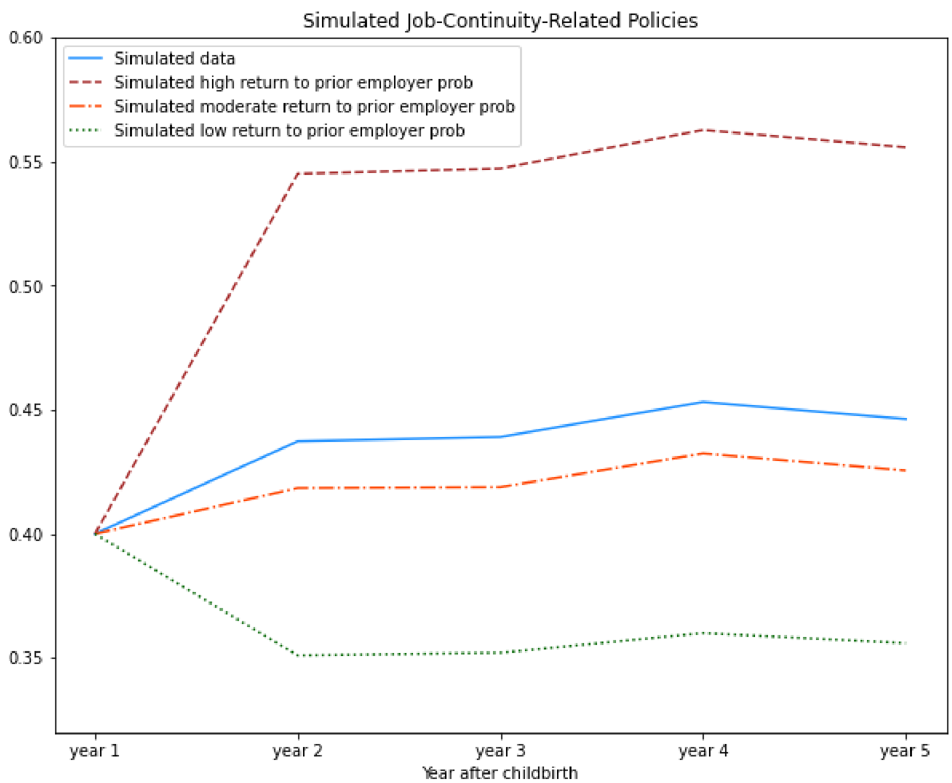

Figure 15: Simulated Job Continuity-Related Policy (full time)

Notes: This figure presents the simulated probability of full-time employment using the parameters estimated by the Dirichlet generalized ordered logit model. In addition, it shows the simulated probability of full-time employment under three counterfactual scenarios, where I hold all other variables unchanged and artificially change the probability of returning to the previous employer for all respondents before simulating full-time employment in the next period: (i) A scenario with a high probability (0.7) of returning to the same employer given they return to work within a year after childbirth, (ii) a scenario with a moderate probability (0.4) of returning to the same employer given they return to work within a year after childbirth, and (iii) a scenario with a low probability (0.1) of returning to the same employer given they return to work within a year after childbirth.

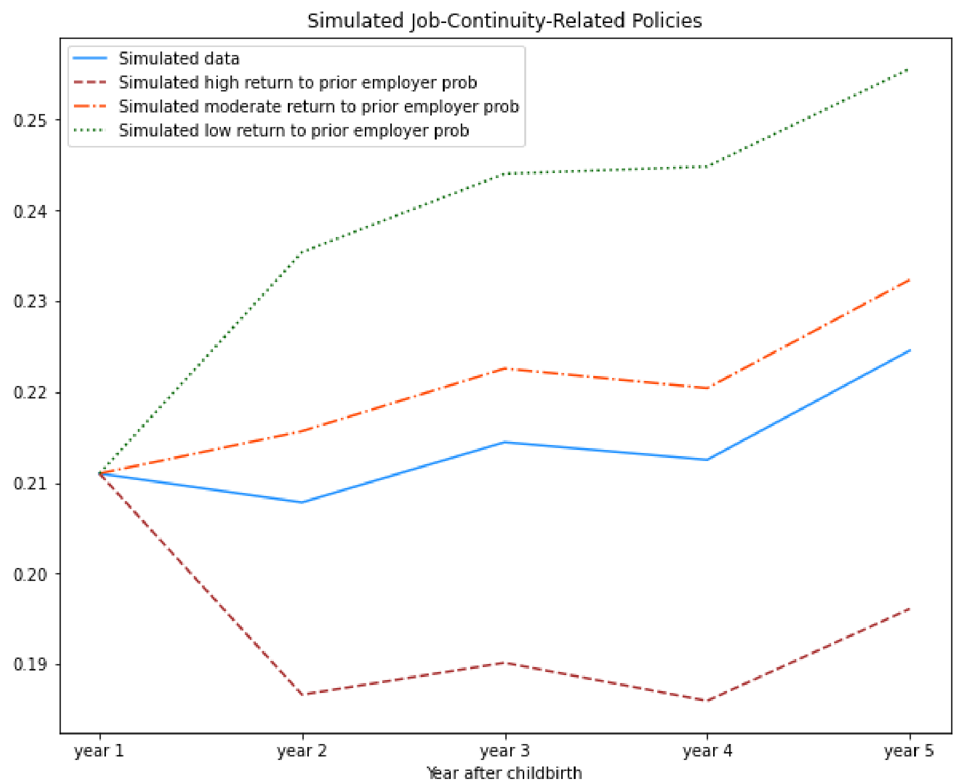

Figure 16: Simulated Job Continuity-Related Policy (non-employment)

Notes: This figure presents the simulated probability of non-employment using the parameters estimated by the Dirichlet generalized ordered logit model. In addition, it shows the simulated probability of full-time employment under three counterfactual scenarios, where I hold all other variables unchanged and artificially change the probability of returning to the previous employer for all respondents before simulating full-time employment in the next period: (i) A scenario with a high probability (0.7) of returning to the same employer given they return to work within a year after childbirth, (ii) a scenario with a moderate probability (0.4) of returning to the same employer given they return to work within a year after childbirth, and (iii) a scenario with a low probability (0.1) of returning to the same employer given they return to work within a year after childbirth.

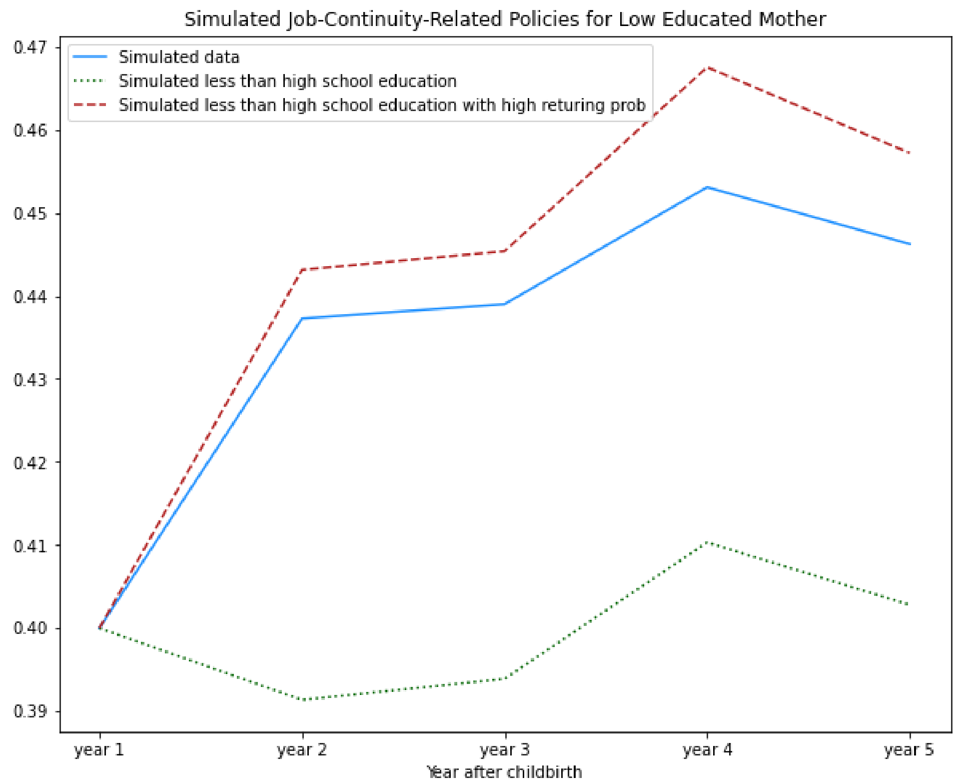

Figure 17: Simulated Job Continuity-Related Policy for Low Educated Mother (full time)

Notes: This figure presents the simulated probability of full-time employment using the parameters estimated by the Dirichlet generalized ordered logit model. In addition, it shows the simulated probability of full-time employment under two counterfactual scenarios, where I hold all other variables unchanged and artificially change the probability of returning to the previous employer for mothers with low education levels before simulating full-time employment in the next period: (i) A scenario for women with low education levels (8 education years), and (ii) A scenario for women with low education levels (8 education years) but having a high probability (0.7) of returning to the same employer given they return to work within a year after childbirth.

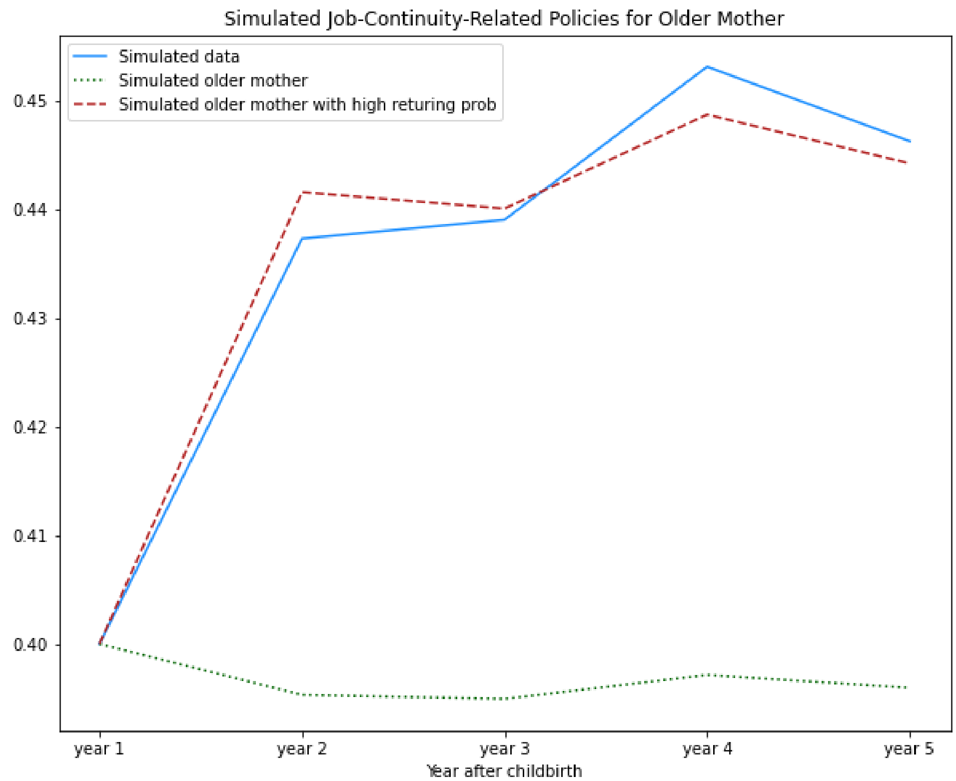

Figure 18: Simulated Job Continuity-Related Policy for Older New Mother (full time)

Notes: This figure presents the simulated probability of full-time employment using the parameters estimated by the Dirichlet generalized ordered logit model. In addition, it shows the simulated probability of full-time employment under two counterfactual scenarios, where I hold all other variables unchanged and artificially change the probability of returning to the previous employer for mothers aged 30 years before simulating full-time employment in the next period: (i) A scenario for 30-year-old women, and (ii) A scenario for 30-year-old women with a high probability (0.7) of returning to the same employer given they return to work within a year after childbirth.

Tables

Table 1: **Definition of Variables**

Variables	**Definition**
Labor-Force Attachment	The labor-force attachment of women, which is divided into three categories: full-time workers (FW), part-time workers (PW), and those who are not working (NW).
Education	Women's education in years.
Age	The age of the women one year after giving birth.
Partner	The dummy variable denotes whether the woman is in a partnership (married or cohabiting).
Log Partner Income	The logarithm of the partner's actual annual income (measured in \$100 units) ($log(patner\ income + 1)$). If the woman is without a partner, the variable is zero.
Log Wage	The logarithm of real hourly wage of the woman (hourly wage in cents).
Religion	The religion preference of the woman, which is divided into four categories: Roman Catholic (baseline), Protestant, other religion and no religion.
Region	The region of the woman, which is divided into four categories: Northeast (baseline), North Central, South and West.

Table 2: **Summary Statistics**

Variable	**N**	**Mean**	**S.D.**
Full-time Workers	12,535	0.438	0.50
Part-time Workers	12,535	0.348	0.48
Not Working	12,535	0.215	0.41
Education	12,535	12.40	2.56
Age	12,535	23.95	4.72
Partner	12,535	0.631	0.48
Log Partner Income	12,535	2.286	2.87
Log Wage	12,535	5.936	2.57
Religion			
Catholic	12,535	0.221	0.41
Protestant	12,535	0.544	0.50
Other Religion	12,535	0.099	0.30
No Religion	12,535	0.136	0.34
Region			
Northeast	12,535	0.147	0.35
North Central	12,535	0.203	0.40
South	12,535	0.433	0.50
West	12,535	0.216	0.41

Table 3: Nested Logit Results

	Not work v.s. Return work	Different employer v.s. Same employer
Constant	0.6373***	-4.9495***
	(0.793)	(0.986)
Education	0.0934**	0.1741***
	(0.038)	(0.039)
Age	-0.1145***	-0.0848**
	(0.029)	(0.037)
Partner	0.7800*	1.2073**
	(0.461)	(0.463)
Log Partner Income	-0.2266**	-0.3093***
	(0.083)	(0.082)
Log Wage	0.1921**	0.2744***
	(0.075)	(0.071)
Education of Women's Mom	-0.0145	-0.0308
	(0.016)	(0.020)
Total Work	-0.0034***	-0.0004
	(0.001)	(0.001)
Percent Work	3.7605***	7.8853***
	(0.384)	(0.507)
Religion		
Protestant	0.0628	0.1318
	(0.169)	(0.187)
Other Religion	-0.2147	-0.1656
	(0.234)	(0.266)
No Religion	-0.0671	-0.4523*
	(0.211)	(0.232)
Region		
North	-0.4523**	-0.3270
	(0.223)	(0.255)
South	-0.3917**	-0.2861
	(0.195)	(0.223)
West	-0.5823**	-0.5969**
	(0.211)	(0.241)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Ordered logit and Generalized Ordered Logit for Job Continuity

	Ordered logit model	Generalized ordered logit model	
		Not work v.s. work	Not full-time v.s. full-time
Prob Return Work	2.728***	2.873***	2.505***
	(0.11)	(0.12)	(0.13)
Prob Same Employer	1.417***	1.482***	1.435***
	(0.06)	(0.10)	(0.06)
cut1	1.033**	-1.139***	
	(0.08)	(0.09)	
cut2	2.841***		-2.658***
	(0.08)		(0.10)
AIC	24111.76	24105.94	
Log likelihood	-12053.88	-12048.97	
N	12535	12535	

Table 5: DGOL for Job Continuity

Variable	Work or Not		Full Time or Not	
	No work	Some Work	Not Full Time	Full Time
Constant	-1.823^{***}	-2.958^{***}	0.784^{***}	-1.866^{***}
	(0.06)	(0.06)	(0.17)	(0.19)
Prob Return Work	-1.525^{***}	1.448^{***}	-0.696^{**}	1.731^{***}
	(0.09)	(0.09)	(0.23)	(0.25)
Prob Same Employer	5.249^{***}	6.488^{***}	-0.911^{***}	0.515^{***}
	(0.07)	(0.07)	(0.12)	(0.13)
AIC	20764.13			
Log likelihood	-10370.06			
N	12535			

Table 6: **Ordered logit for Job Continuity and Women Characteristics**
(standard deviations in parentheses)

Variable	Coefficient
Prob Return Work	2.328***
	(0.12)
Prob Same Employer	1.568***
	(0.07)
Education	0.089***
	(0.01)
Age	-0.052***
	(0.01)
Partner	-0.215
	(0.13)
Log Partner Income	0.018
	(0.02)
Log Hourly Wage	0.041
	(0.02)
Religion	
Protestant	-0.041
	(0.05)
Other Religion	0.084
	(0.07)
No Religion	0.047
	(0.06)
Region	
North	-0.003
	(0.06)
South	0.113**
	(0.05)
Wes	0.028
	(0.06)
cut1	0.877**
	(0.20)
cut2	2.698***
	(0.21)
AIC	24004.58
Log likelihood	-11987.29
N	12535

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: **Generalized Ordered Logit for Job Continuity and Women Characteristics**
(standard deviations in parentheses)

Variable	Not work v.s. work	Not full-time v.s. full-time
Prob Return Work	2.465***	2.355***
	(0.15)	(0.15)
Prob Same Employer	2.027***	1.274***
	(0.10)	(0.07)
Education	0.077***	0.091***
	(0.01)	(0.01)
Age	−0.096***	−0.015**
	(0.01)	(0.01)
Partner	−0.216	−0.214
	(0.18)	(0.14)
Log Partner Income	−0.000	0.031
	(0.03)	(0.03)
Log Hourly Wage	0.006	0.028
	(0.03)	(0.03)
Religion		
Protestant	0.050	−0.097
	(0.06)	(0.05)
Other Religion	0.186	0.039
	(0.08)	(0.07)
No Religion	0.203	−0.055
	(0.08)	(0.07)
Region		
North	−0.005	−0.014
	(0.08)	(0.07)
South	0.068	0.140*
	(0.07)	(0.06)
West	0.090	0.004
	(0.08)	(0.07)
Constant	0.417	−3.456***
	(0.25)	(0.24)
AIC	23712.30	
Log likelihood	-11826.15	
N	12535	

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: **DGOL for Job Continuity and Women Characteristics**
(standard deviations in parentheses)

Variable	Work or Not		Full Time or Not	
	No work	Some Work	Not Full Time	Full Time
Prob Return Work	-1.311***	1.234***	-0.763***	1.518***
	(0.06)	(0.06)	(0.24)	(0.27)
Prob Same Employer	4.974***	6.763***	-0.391**	0.885***
	(0.06)	(0.06)	(0.13)	(0.15)
Education	0.041***	0.120***	-0.083***	0.009
	(0.01)	(0.01)	(0.02)	(0.02)
Age	-0.064***	-0.160***	-0.041***	-0.056***
	(0.01)	(0.01)	(0.01)	(0.01)
Partner	-0.064	-0.289***	0.294***	0.215
	(0.05)	(0.05)	(0.10)	(0.13)
Log Partner Income	-0.270***	-0.264***	-0.072***	-0.066**
	(0.01)	(0.01)	(0.02)	(0.03)
Log Hourly Wage	-0.294***	0.279***	-0.042	0.032
	(0.02)	(0.02)	(0.04)	(0.05)
Religion				
Protestant	-0.019	-0.007	0.058	-0.043
	(0.04)	(0.04)	(0.08)	(0.08)
Other Religion	-0.094	0.007	0.031	0.063
	(0.05)	(0.05)	(0.10)	(0.11)
No Religion	0.064	0.182***	0.224**	0.152
	(0.05)	(0.05)	(0.10)	(0.12)
Region				
North	0.285***	0.225***	-0.035	-0.109
	(0.04)	(0.04)	(0.10)	(0.11)
South	-0.290***	-0.254***	-0.172*	-0.049
	(0.03)	(0.03)	(0.10)	(0.09)
West	-0.051	-0.001	-0.163	-0.188*
	(0.04)	(0.04)	(0.11)	(0.11)
Constant	-2.677***	-2.103***	3.241***	-0.071
	(0.11)	(0.11)	(0.39)	(0.45)
AIC	20608.32			
Log likelihood	-10240.16			
N	12535			

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9: **Average Partial Effects**

	APE	
	Actual	Simulated
$P(y_t = \text{full time} \vert y_{t-1} = \text{full time})$	0.794	0.839
$P(y_t = \text{full time} \vert y_{t-1} = \text{not full time})$	0.181	0.191
$P(y_t = \text{full time} \vert y_{t-1} = \text{full time}) - P(y_t = \text{full time} \vert y_{t-1} = \text{not full time})$	0.613	0.648